SOME ASPECTS OF THE PROPERTY OF ANGLE OF REPOSE OF POWDERS

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Four methods of determining the angle of repose of free flowing powders have been critically compared using graded samples of glass balls, lead shot and silver sand. The method used influenced the result. Results of all methods have been correlated graphically and values for static and dynamic interparticulate friction have been assessed for the materials used. It was found that results for spheres could be correlated on one graph, irrespective of type of material and an explanation is given to account for this.

AN intrinsic property of any powder is its resistance to differential movement between particles when subjected to external forces. Probably this property should be best described as *interparticulate friction*, of which the *angle of repose* is a manifestation. Practical methods[1–7] of assessment of interparticulate friction are mainly based on the measurement of the angle of repose of the loose powder mass. There would appear to be four main methods of measuring the angle of repose and, for comparison, the essentials of these methods are set out in the line diagrams in Figure 1.

Method I. Fixed Funnel and Free Standing cone[1–4]

A funnel with the end of the stem cut perpendicular to the axis of symmetry is secured with its tip a given height, H, above graph paper placed on a flat horizontal surface. Powder is carefully poured through the funnel until the apex of the conical pile so formed just reaches the tip of the funnel. The mean diameter, 2R, of the base of the powder cone is determined and the tangent of the angle of repose is given by $\tan \alpha = \frac{H}{R}$, where α = angle of repose.

Method II. Fixed Bed Cone[5]

The diameter of the base is fixed by using a circular dish with sharp edges, or a suitably machined container. Powder is poured on to the centre from a funnel which can be raised vertically until a maximum cone height, H, is obtained, $\tan \alpha$ being calculated as before.

Method III. Tilting Box[4]

A rectangular box is filled with powder and tipped until the contents begin to slide. The angle which the upper surface of the box makes with the horizontal is taken as the angle of repose.

Method IV. Revolving Cylinder[6–7]

A sealed hollow cylinder with one end transparent is made to revolve horizontally. It is half-filled with the powder, so that the free surface of the powder forms a diametrical plane. The maximum angle that this plane makes with the horizontal on rotation of the container is taken as the angle of repose.

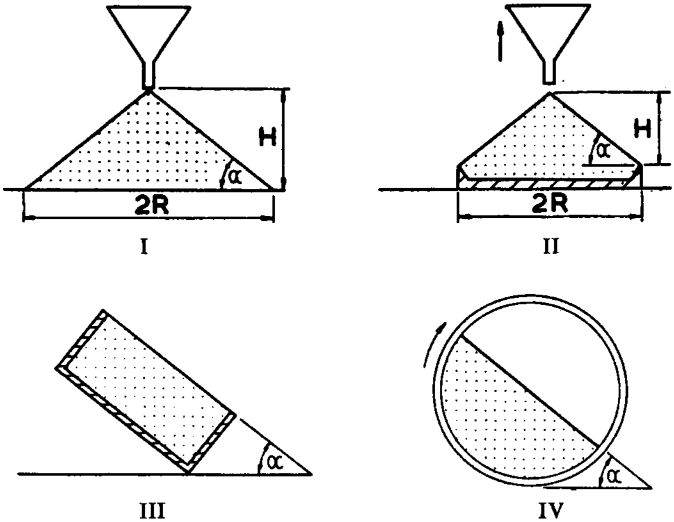

FIG. 1. Four main methods of measuring the angle of repose.
I. Fixed funnel and free standing cone.
II. Fixed bed cone.
III. Tilting box.
V. Revolving cylinder.

No published investigation has been found in which the methods are simultaneously compared, yet an inspection of reported results indicates a range in the angles of repose for what would appear to be the same material, e.g., lead shot, silver sand, or mustard seed. A critical examination of the methods indicated in Figure 1 has been made as a possible means of assessing this property.

EXPERIMENTAL AND RESULTS

All materials were graded by sieving in order to reduce complications introduced by separation during the practical runs. The size ranges of the powders used are reported in terms of the numbers of mesh of the B.S. sieves[8] firstly through which the particles will pass and secondly on which they will be retained (e.g., 60/80 powder). Simple free flowing materials were used in this series of experiments to ensure that complications due to shape and surface characteristics of the particles would be reduced to a minimum. The materials were allowed to reach equilibrium under conditions of 68° F. and 40 per cent relative humidity before the experiments were carried out.

Glass balls and lead shot. These two materials were chosen because they have a similar shape but differing specific gravities (glass, 2·2; lead, 11·3) and surface friction characteristics.

Sand. This was chosen because it had a characteristic shape, and a wide range of sizes were easily obtainable. Silver sand was wet sieved, dried and resieved to ensure reproducible grading.

Test of Method I. The technique described by Neumann[2] was followed using a funnel with a stem bore of 0·3 cm. diameter with the tip of the stem 2·0 cm. above the surface. But, it was noticed that the heap often collapsed before the top of the cone reached the tip of the funnel; this sort of slip gave an erroneously large value for the base diameter and hence a low value for the angle of repose. The incidence of experiments in which the heap collapsed is greater if the diameter of the stem bore is increased and, with any given stem diameter, if the rate at which powder is added to the heap is increased. With the largest particles a shallow cone resulted and this led to an investigation of the effect of varying the height of the stem tip above the horizontal surface. Experiments were also made in which the particles were not added to the heap using the funnel, but were added through a glass tube, of which one end was drawn out to a narrow tip, held at a shallow angle so that the flow of particles was under complete control. A selection of results is given in Table I.

TABLE I

MAXIMUM ANGLE OF REPOSE MEASUREMENTS USING A FUNNEL AT A FIXED HEIGHT ABOVE SURFACE

Material	Glass spheres		Lead shot		Silver sand	
Grading, mesh ..	100/120		8/10		60/80	
Conditions	A	B	A	B	A	B
Height of cone apex above surface, cm.	Tangent of angle of repose $\times 10^3$					
1·0	580	630	520	540	880	960
2·0	550	605	500	580	830	930
3·0	500	550	490	545	790	800
4·0	465	520	465	520	730	770
5·0	465	512	450	515	690	760
6·0	435	510	420	510	650	690

A. Maximum result obtained using Neumann's technique.
B. Maximum result obtained by careful addition of powder through a narrow inclined tube.

Column A represents the maximum result when the powder was poured through a funnel following Neumann's technique; column B represents the maximum result when all precautions are made to reduce particle momentum to a minimum. It was noticed with all materials that there was an occasional collapse of the heap apparently due to the material slipping on the paper surface. This effect was considerably reduced by using a horizontal surface of coarse sand paper but its use made the measurement of the base diameter of the heap difficult.

Test of Method II. Nelson's technique[5] was followed initially, but the momentum with which the particles hit the pile greatly increased the incidence of heap collapse. This observation led to an investigation of the best methods by which a cone could be made by carefully building up the pile by the technique described above. The results in Table II indicate the range which can be achieved between the published technique (column A) and the alternative method (column B). The effect of varying the diameter of the base was investigated, and also, when the heap collapsed, the angle of repose of the surface of the remaining slope was measured (column C).

TABLE II

MAXIMUM ANGLE OF REPOSE MEASUREMENTS USING FIXED BED METHOD

Material	Glass spheres			Lead shot			Silver sand		
Grading, mesh	14/16			16/22			16/22		
Conditions	A	B	C	A	B	C	A	B	C
Radius of base, cm.	Tangent of angle of repose × 10³								
0·64	560	650	—	560	630	—	840	960	—
1·28	558	597	480	560	600	460	800	920	700
1·80	522	557	465	560	580	480	785	870	650
2·48	517	544	445	530	560	445	775	835	650
3·43	490	554	425	510	550	430	770	810	630
4·38	485	520	420	500	530	420	740	780	660
7·01	485	515	425	490	510	415	725	750	650
11·00	460	520	430	470	510	420	690	710	650

A. Maximum result using Nelson's technique.
B. Maximum result obtained by careful addition of powder through a narrow inclined tube.
C. Residual slope of heap measured after slip had taken place.

Test of Method III. Takahasi[4] did not indicate the size of box he used, but preliminary experiments using a 30/44 mesh grading of glass balls and various shaped boxes at hand indicated that the dimensions of the bed were of critical importance. The use of boxes was not satisfactory and a tilting table was devised (Figure 2) consisting of a flat plate (N) mounted

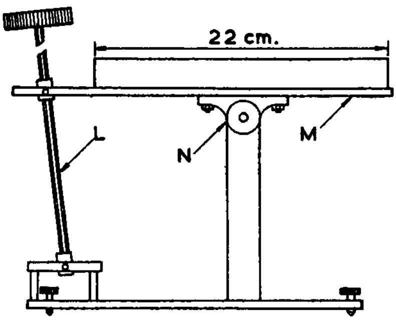

FIG. 2. Tilting table.

on standard pulley blocks (M) so that it was free to turn about a horizontal axis. Side pieces were mounted to this base plate to form the equivalent of three sides of a box. Movement was precisely controlled by the screwed rod (L). In use it was found that, to reduce base and wall effects, the depth of the bed should be at least 20 particle diameters of mean particle size (2 cm. depth is suitable for powders) and the width should be not less than one-third of the length of the bed. It was also found that a layer of coarse sand paper along the bottom of the bed was necessary to prevent preferential slip of the particles along this plane. The material was placed as a bed of given length and the excess cleaned off to give a surface flush with the top of the table frame. The bed was then carefully tilted until slip occurred. The limiting angle at which this occurred was taken as the angle of repose. A selection of results is given in Table III.

TABLE III

MAXIMUM ANGLE OF REPOSE USING TILTING TABLE

Material	Glass spheres		Lead shot		Silver sand	
Grading, mesh ..	100/120	14/16	16/22	8/10	100/120	16/22
Length of bed, cm.	Tangent of angle of repose × 10^3					
1·0	740	—	680	—	1010	—
2·0	760	780	720	—	960	880
3·0	680	640	660	620	920	940
5·0	630	650	620	630	850	830
7·0	615	625	630	630	780	800
10·0	580	600	570	595	760	760
14·0	570	570	550	560	740	725
18·0	550	550	570	550	725	730
22·0	560	540	550	540	715	725

Test of Method IV. Cylinders of various diameters were used. The curved walls were lined with sand paper in order to prevent preferential slip between the powder and the walls of the container. A selection of results is given in Table IV.

TABLE IV

MAXIMUM ANGLE OF REPOSE USING REVOLVING CYLINDER

Material	Glass spheres		Lead shot		Silver sand	
Grading, mesh ..	100/120	14/16	16/22	8/10	100/120	16/22
Diameter of cylinder, cm.	Tangent of angle of repose × 10^3					
2·0	750	760	740	660	900	920
5·5	615	640	615	630	800	780
9·0	590	580	560	585	760	740
12·0	570	580	550	570	725	735

DISCUSSION

Inspection of the results indicates that, for any given method, the magnitude of the angle of repose falls as the size of the heap or bed increases. Direct comparison between methods is difficult because the basis for measurement of the size of the powder heap or bed varies from one method to another, although it is clear that methods I and II give results that are lower than those given by methods III and IV. This difficulty in comparison has been overcome by presenting the results in graphical form as in Figures 3 and 4. In these graphs the abscissa represents the horizontal base and a line joining a given point and the origin makes the angle of repose with this datum line.

All the results in each graph lie within a well defined zone, the lower boundary of which is the locus of the points recording the natural angle of slip (the results in Table II, column C) and the upper boundary of which is the locus of points obtained when a flat surface of the material is tilted until it does slip. These boundaries tend to straight lines for larger measurements, but extrapolation of the straight portion of each line does not pass through the origin.

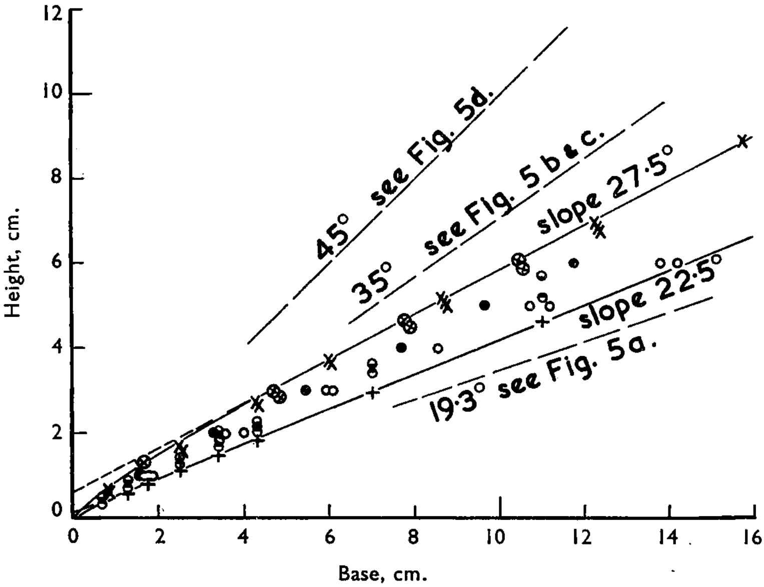

FIG. 3. Combined results for glass balls and lead shot of experimental angle of repose measurements. Key below Fig. 4.

For friction between flat surfaces, it is well established that the coefficient of static friction has a larger value than the coefficient of kinetic friction. With the present data, the angle of the slope of the linear portion of the upper boundary line represents the static angle of repose whilst the angle of the slope of the lower boundary line represents the dynamic angle of repose. This is in agreement with the observations of Franklin and Johanson[6] and of Pridham[7].

In practice, the position which an experimental result will occupy between the boundaries will depend on the technique adopted to measure the angle of repose, but all the data of this work (only a representative selection has been included in the Tables) confirm that results for a given material using a set procedure are reproducible. With the heaped cone techniques, the magnitude of the final ratio of height to base depends on reducing the momentum of the particles (otherwise the stability of the existing heap is upset and general slip takes place) and also in forming the heaps in such a way that in *all* sectors the slope is built up to the limiting angle (which, however, must not be exceeded, otherwise premature slip takes place). These are practical difficulties and, consequently, results tend to be low.

Figure 4 gives the results for silver sand. No size effect was noted except that for small heaps or beds, wall and end effects gave erratic results.

All the results for spheres have been included in Figure 3 irrespective of size or type of material. It was observed that failure in the case of an assemblage of equally sized spheres of a given material was mainly by rolling of the added particles down the slope of the heap. This led to a

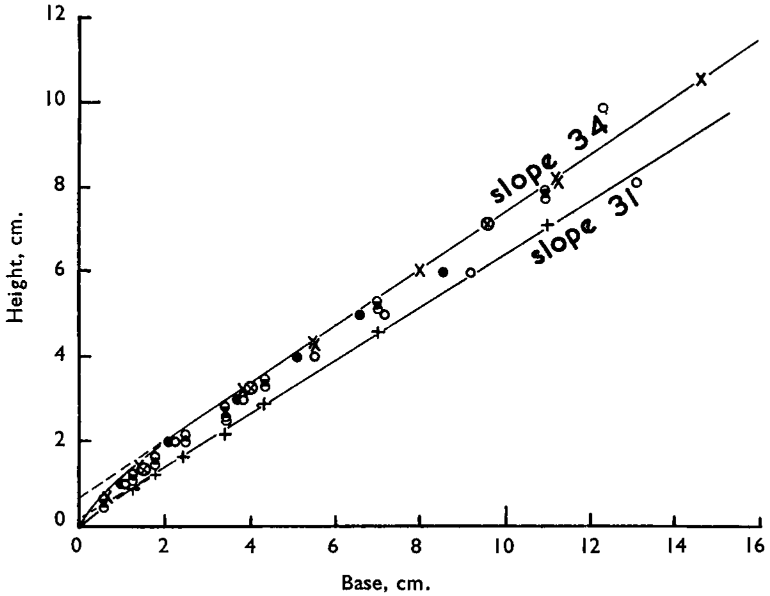

FIG. 4. Results for angle of repose measurements for silver sand.

Key to Figs. 3 and 4.

- ○ Method I, Neumann's technique (Table I, A).
- ● Method I, using narrow inclined tube (Table I, B).
- ◒ Method II, Nelson's technique (Table II, A).
- ◓ Method II, using narrow inclined tube (Table II, B).
- \+ Method II, Residual slope of heap after slip (Table II, C).
- × Method III, Tilting table (Table III).
- ⊗ Method IV, Revolving cylinder (Table IV).

consideration of the condition which would allow a particle to roll down the slope of randomly packed spheres of the same size, and it was found to depend mainly on the type of packing in the surface. A sphere will stay in a hollow formed by a grid of three or more spheres on a slope providing its centre of gravity does not fall outside the lower boundary of the grid. The closest packing on a regular triangular grid permits rolling if the slope of the plane is 19·3° ($\tan^{-1}$ 0·351) for one axis of symmetry (Figure 5*a*) and 35·1° ($\tan^{-1}$ 0·702) for the alternative axis (Figure 5*b*). In the case of regular square packing, the necessary slopes to permit rolling are 35·3° ($\tan^{-1}$ 0·709, Figure 5*c*) and 45° ($\tan^{-1}$ 1·0, Figure 5*d*). Thus for regular packings, the smallest angle for a slope on which rolling will take place is 19·3° and the largest possible angle which can prevent movement is 45°. Plane slopes were made using in turn samples of glass spheres, lead shot, $\frac{1}{4}$ in. steel ball bearings, and also table-tennis balls and in all cases these limiting conditions were confirmed. In practice, however, a surface of a heap will consist of a random selection of these grids and the slope will have an angle similar to those given in Figure 3 where the slope of the lower boundary is 22·5° ($\tan^{-1}$ 0·414) and of the upper boundary is 27·5° ($\tan^{-1}$ 0·521).

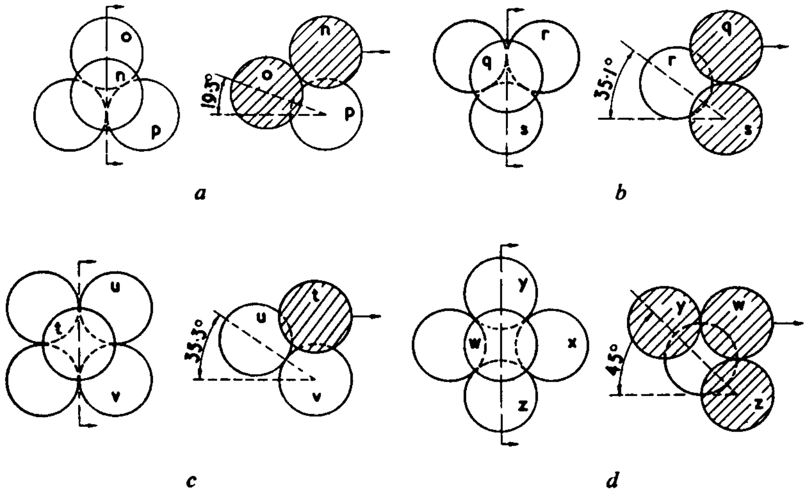

FIG. 5. Limiting conditions for rolling of a ball supported on sloping grids of 3 or 4 balls. See text for further explanation.

Recently, Brown and Richards[9] presented a paper to the Institution of Chemical Engineers describing yet another variation of angle of repose measurement. A circular platform of known diameter was immersed in the centre of a large container filled with the particulate material, which was allowed to escape slowly from the bottom of the container. The height of the pile remaining on the platform was measured to calculate the angle of repose. It is interesting that under these conditions the angle of repose for glass balls is reported as 18·9° and 20·7°. The method of Brown and Richards probably simulates the best conditions to measure interparticulate friction within a *moving* bed of particles and therefore, on the findings of my work, measures the dynamic angle of repose corresponding to the lower boundary slope in Figure 3.

CONCLUSION

Most methods of angle of repose measurements will provide the necessary data to allow suitable comparison between samples during routine quality control tests. However, in order to ensure reproducibility between workers or laboratories it is necessary to define rigid practical conditions.

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